Answers Exercise 2 Stochastic Models of Manufacturing Systems 4T400, 21 April

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1. You toss an *unfair* coin twice; the probability of Head is 0.3. Compute the conditional probability that the outcome of the second toss is Head, given that the outcome of both tosses is different.

   Answer: $\frac{1}{2}$

2. Each member of a family of 4 persons writes his name on a piece of paper, and these pieces of papers are randomly divided among the family members. What is the probability that each person gets a piece of paper with the name of someone else?

   Answer: $\frac{9}{24}$

3. The radius of a circle is random between 0 and 6 meter. Compute the expected area of the circle.

   Answer: 12π

4. A batch consists of exactly 10 products, but the mix of products is random. With probabilty 0.1 a product is of type A, otherwise it is of type B. Compute the probability that the batch contains exactly two type A products?

   Answer: $\binom{10}{2}0.1^2 0.9^8 = 0.1937$

5. Consider the batch from the previous question. The expected processing time of a type A product is 2 minutes, and this is 10 minutes for a type B product. What is the expected processing time of a complete batch?

   Answer: 92 minutes

6. The lifetime of two light bulbs are independent and exponentially distributed. The mean lifetime of bulb A is 20 hours, and it is 40 hours for bulb B. At $t = 0$ both bulbs are working. What is the probability that both bulbs are still working after 20 hours?

   Answer: $e^{-\frac{3}{2}} = 0.223$

7. Consider the two light bulbs from the previous question. Calculate the expected time till one of the two bulbs breaks down.

   Answer: $\frac{40}{3} = 13\frac{1}{3}$ hours

8. A magician asks you to guess the card that he is holding, which he has blindly drawn from a deck of 52 cards. But before you make a guess, you may ask one question. Which question maximizes the probability of success?

   A. Is the card you are holding black?

   B. Is the card you are holding two of diamonds?

   C. It doesn't matter whether you ask question A or B.

   Answer: C

   (a) Calculate the constant c.
   $$\int_0^\infty cxe^{-\frac{1}{2}x}dx = 4c = 1,$$
   so $c = \frac{1}{4}$

(b) What is the probability that the pick-and-place machine has no failures within 4 hours?

$$P(X > 4) = \int_4^\infty \frac{1}{4} x e^{-\frac{1}{2}x} dx = 3e^{-2} = 0.406$$

(c) Calculate the expected time to failure.

$$E(X) = \int_0^\infty \frac{1}{4} x^2 e^{-\frac{1}{2}x} dx = 4 \text{ uur}$$